ÉRETTSÉGI VIZSGA • 2017. május 9.

# MATEMATIKA ANGOL NYELVEN

# EMELT SZINTŰ ÍRÁSBELI VIZSGA

# JAVÍTÁSI-ÉRTÉKELÉSI ÚTMUTATÓ

EMBERI ERŐFORRÁSOK MINISZTÉRIUMA

# Instructions to examiners

**Formal requirements:**

1. Mark the paper **legibly, in ink, different in colour** from that used by the candidate.
2. The first of the rectangles next to each problem shows the maximum attainable score on that problem. The **score** given by the examiner is to be entered **into the rectangle** next to this.
3. **If the solution is perfect**, enter maximum score and, with a checkmark, indicate that you have seen each item leading to the solution and consider them correct.
4. If the solution is incomplete or incorrect, please **mark the error** and also indicate the individual **partial scores**. It is also acceptable to indicate the points lost by the candidate if it makes grading the paper easier. After correcting the paper, it must be clear about every part of the solution whether that part is correct, incorrect or unnecessary.
5. Please, **use the following symbols** when correcting the paper:
   - correct calculation: *checkmark*
   - principal error: *double underline*
   - calculation error or other, not principal, error: *single underline*
   - correct calculation with erroneous initial data: *dashed checkmark or crossed checkmark*
   - incomplete reasoning, incomplete list, or other missing part: *missing part symbol*
   - unintelligible part: *question mark* and/or *wave*
6. Do not assess anything written **in pencil**, except for diagrams

**Assessment of content:**

1. The answer key may contain more than one solution for some of the problems. If the **solution given by the candidate is different**, allocate the points by identifying parts of the solution equivalent to those given in the answer key.
2. Subtotals may be **further divided, unless stated otherwise in the answer key**. However, scores awarded must always be whole numbers.
3. If there is a **calculation error** or inaccuracy in the solution, take points off only for the part where the error occurs. If the reasoning remains correct and the error is carried forward while the nature of the problem remains unchanged, points for the rest of the solution must be awarded.
4. **In case of a principal error**, no points should be awarded at all for that section of the solution, not even for steps that are formally correct. (These logical sections of the solutions are separated by double lines in the answer key.) However, if the erroneous information obtained through principal error is carried forward to the next section or the next part of the problem, and it is used there correctly, the maximum score is due for that part, provided that the error has not changed the nature of the task to be completed.
5. Where the answer key shows a **unit** or a **remark** in brackets, the solution should be considered complete without that unit or remark as well.

6. If there are more than one different approach to a problem, **assess only the one indicated by the candidate**. Please, mark clearly which attempt was assessed.
7. **Do not give extra points** (i.e. more than the score due for the problem or part of problem).
8. The score given for the solution of a problem, or part of a problem, **may never be negative**.
9. **Do not take points off** for steps or calculations that contain errors but are not actually used by the candidate in the solution of the problem.
10. **The use of calculators** in the reasoning behind a particular solution **may be accepted without further mathematical explanation in case of the following operations**: addition, subtraction, multiplication, division, calculating powers and roots, $n!$, $\binom{n}{k}$, replacing the tables found in the 4-digit Data Booklet (sin, cos, tan, log, and their inverse functions), approximate values of the numbers $\pi$ and $e$, finding the solutions of the standard quadratic equation. No further explanation is needed when the calculator is used to find the mean and the standard deviation, as long as the text of the question does not explicitly require the candidate to show detailed work. **In any other cases, results obtained through the use of a calculator are considered as unexplained and points for such results will not be awarded.**
11. Using **diagrams** to justify a solution (e.g. reading data off diagrams by way of direct measurement) is unacceptable.
12. **Probabilities** may also be given in percentage form (unless stated otherwise in the text of the problem).
13. If the text of the problem does not instruct the candidate to round their solution in a particular way, any solution **rounded reasonably and correctly** is acceptable even if it is different from the one given in the answer key.
14. **Assess only four out of the five problems in part II of this paper**. The candidate was requested to indicate in the appropriate square the number of the problem not to be assessed and counted towards their total score. Should there be a solution to that problem, it does not need to be marked. However, if the candidate did not make a selection and neither is their choice clear from the paper, assume automatically that it is the last problem in the examination paper that is not to be assessed.

# I.

| **1. a)** | | |
|---|---|---|
| As the log function is strictly increasing, $x < 100$. | 1 point | *These points are also due for a correct diagram.* |
| (From the domain of the log function:) $x > 0$. | 1 point | |
| The solution: $0 < x < 100$. | 1 point | *The solution set is* ]0; 100[ |
| **Total:** | **3 points** | |

| **1. b)** | | |
|---|---|---|
| In standard form: $x^2 + 4x - 5 < 0$. | 1 point | |
| The solutions of the equation $x^2 + 4x - 5 = 0$ are 1 and –5. | 1 point | |
| As the quadratic coefficient is positive, | 1 point | *This point is also due for a correct diagram or for factoring into* $(x + 5)(x - 1) < 0$ *form.* |
| so $-5 < x < 1$. | 1 point | |
| **Total:** | **4 points** | |

| **1. c)** | | |
|---|---|---|
| $0.5^{\vert x-3\vert} < 0.5^2$ | 1 point | |
| The base 0.5 exponential function is strictly decreasing. | 1 point | |
| $\vert x - 3\vert > 2$. | 1 point | |
| This happens if either $x - 3 > 2$ or $x - 3 < -2$, | 1 point | |
| so $x > 5$ or $x < 1$. | 1 point | |
| **Total:** | **5 points** | |

*Note: Award a maximum of 3 points if the candidate solves the inequality* $|x - 3| < 2$.

| **2. a)** | | |
|---|---|---|
| (Assuming Noémi receives $x$ percent on her oral presentation) the final percentage of the exam is calculated as $\frac{2 \cdot 73 + 5 \cdot 64 + 3x}{2 + 5 + 3}$. | 1 point | *This point is also due if the correct reasoning is reflected only by the solution.* |
| According to the text of the question: $\frac{2 \cdot 73 + 5 \cdot 64 + 3x}{2 + 5 + 3} \geq 70$. | 1 point | |
| $x \geq 78$ | 1 point | |
| Noémi needs to receive at least 78% on her oral presentation. | 1 point | |
| **Total:** | **4 points** | |

*Note: Award maximum score if the candidate finds the correct answer by solving an equation instead of an inequality.*

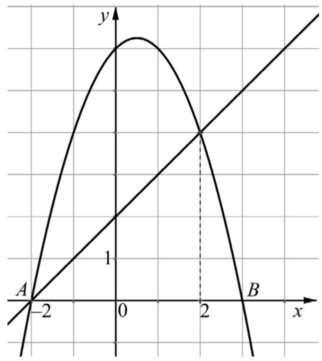
| **2. b)** **Solution 1** | | |
|---|---|---|
| (Assuming there are $n$ first-year students) the number of boys is $n-75$, the number of students not staying at the university hall is $n-40$. | 1 point | *These points are also due if the correct reasoning is reflected only by the solution.* |
| The average on the one hand is $\frac{75\cdot 70+(n-75)\cdot 62}{n}$ and also on the other hand it is $\frac{40\cdot 71+(n-40)\cdot 65}{n}$. | 1 point | |
| The equation $\frac{75\cdot 70+(n-75)\cdot 62}{n}=\frac{40\cdot 71+(n-40)\cdot 65}{n}$ is to be solved. | 1 point | |
| $n=120$ | 2 points | *These 2 points are due for solving the equation.* |
| There are 120 first-year students taking the exam. | 1 point | |
| Checking against the original text (the average of all 120 students is 67% in both cases). | 1 point | |
| **Total:** | **7 points** | |

| **2. b)** **Solution 2** | | |
|---|---|---|
| Let $f$ be the number of boys, and $k$ the number of students staying at the university hall. In this case the average result is $\frac{75\cdot 70+f\cdot 62}{f+75}$ on the one hand, and also $\frac{40\cdot 71+k\cdot 65}{k+40}$ on the other. | 1 point | *This point is also due if the correct reasoning is reflected only by the solution.* |
| According to the text of the question: $f+75=k+40$, | 1 point | |
| and $\frac{75\cdot 70+f\cdot 62}{f+75}=\frac{40\cdot 71+k\cdot 65}{k+40}$. | 1 point | |
| $f=45$ (and $k=80$). | 2 points | *These 2 points are due for solving the equation system.* |
| There is a total of (75 + 45 =) 120 students taking the exam. | 1 point | |
| Checking against the original text (the average of all 120 students is 67% in both cases). | 1 point | |
| **Total:** | **7 points** | |

| **3. a)** | | |
|---|---|---|
| The median is 83.5 (kg), | 1 point | |
| the mean is 79.75 (kg), | 1 point | |
| the standard deviation is $\sqrt{\frac{2.25^2+5.75^2+10.25^2+8.25^2+2\cdot 5.25^2+16.75^2+8.75^2}{8}}=$ | 1 point | *This point is also due if the candidate uses a calculator and gives the correct answer without further explanation.* |
| $(=\sqrt{77.9375})\approx 8.83$ (kg). | 1 point | |
| **Total:** | **4 points** | |

| **3. b)** | | |
|---|---|---|
| As 90 + 88 = 178, | 1 point | |
| 85 + 82 + 63 = 230 and<br>85 + 71 + 74 = 230, | 1 point | |
| it can really be done in 3 rounds. | 1 point | |
| **Total:** | **3 points** | |

| **3. c)** | | |
|---|---|---|
| (The total mass of any three people is less than 300 kg.) One possibility is 2 people per round (in 4 rounds), or 2 people in one round and 3-3 people in two more rounds (3 rounds total). | 1 point | *This point is also due if the correct reasoning is reflected only by the solution.* |
| In case of two people per round in 4 rounds the number of different possibilities is:<br>$\binom{8}{2}\cdot\binom{6}{2}\cdot\binom{4}{2}\cdot\binom{2}{2}$ (= 2520). | 1 point | *Arrange the 8 people in a line. Order within the 4 groups of two is not considered. The number of possibilities is* $\frac{8!}{(2!)^4}$ (= 2520). |
| Possible arrangements in case of three rounds:<br>2 + 3 + 3, or 3 + 2 + 3, or 3 + 3 + 2. | 1 point | *This point is also due if the correct reasoning is reflected only by the solution.* |
| Each case is possible in $\binom{8}{2}\cdot\binom{6}{3}\cdot\binom{3}{3}$ (= 560) different ways. | 1 point | $\frac{8!}{2!\cdot(3!)^2}$ |
| so, there are $3\cdot\binom{8}{2}\cdot\binom{6}{3}\cdot\binom{3}{3}$ (= 1680) different possibilities. | 1 point | |
| The total number of different possibilities is then<br>$\binom{8}{2}\cdot\binom{6}{2}\cdot\binom{4}{2}\cdot\binom{2}{2}+3\cdot\binom{8}{2}\cdot\binom{6}{3}\cdot\binom{3}{3}=$ | 1 point | |
| = 4200. | 1 point | |
| **Total:** | **7 points** | |

| **4. a)** | | |
|---|---|---|
| The points of intersection between the two graphs are obtained by solving the equation system $\left.\begin{array}{l} y=-x^2+x+6 \\ 0=x-y+2 \end{array}\right\}$. | 1 point |  |
| Express $y$ from the second equation: $-x^2+x+6=x+2$ | 1 point | |
| (Rearrange: $x^2=4$ ) $x_1=-2$ and $x_2=2$. | 1 point | |
| (As the parabolic arc is above the line over the interval [–2; 2]) $T=\int_{-2}^{2}\left((-x^2+x+6)-(x+2)\right)dx=$ | 1 point* | |
| $=\int_{-2}^{2}(-x^2+4)dx=$ | 1 point* | |
| $=\left[-\frac{x^3}{3}+4x\right]_{-2}^{2}=$ | 1 point* | |
| $=\left(-\frac{2^3}{3}+4\cdot 2\right)-\left(-\frac{(-2)^3}{3}+4\cdot(-2)\right)=\frac{16}{3}-\left(-\frac{16}{3}\right)=$ | 1 point* | |
| $=\frac{32}{3}$ | 1 point* | |
| **Total:** | **8 points** | |

*Note: The 5 points marked by * may also be given for the following reasoning:*

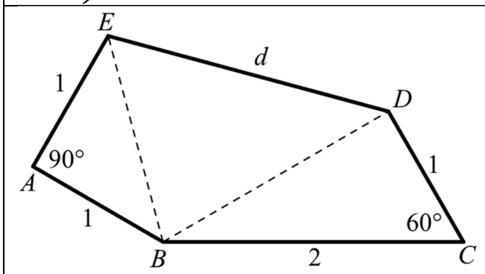
| | | |
|---|---|---|
| $\int_{-2}^{2}(-x^2+x+6)dx=\left[-\frac{x^3}{3}+\frac{x^2}{2}+6x\right]_{-2}^{2}=$ | 1 point | |
| $=\frac{34}{3}-\left(-\frac{22}{3}\right)=\frac{56}{3}$ | 1 point | |
| $\int_{-2}^{2}(x+2)dx=\left[\frac{x^2}{2}+2x\right]_{-2}^{2}=$ | 1 point | |
| = 6 – (–2) = 8 | 1 point | |
| (As the parabolic arc is above the line over the interval [–2; 2]) $T=\frac{56}{3}-8=\frac{32}{3}$. | 1 point | |

| **4. b)** | | |
|---|---|---|
| At the point of intersection on the $x$-axis $y = 0$, and so $-x^2 + x + 6 = 0$. | 1 point | |
| $x_1 = -2$, $x_2 = 3$. | 1 point | |
| Since the first coordinate of $B$ is positive, it is $B(3; 0)$. | 1 point | |
| The derivative of the function $f(x) = -x^2 + x + 6$ ($x \in \mathbf{R}$) is $f'(x) = -2x + 1$ ($x \in \mathbf{R}$). | 1 point* | |
| The gradient of the tangent line drawn at point $B$ is $f'(3) =$ | 1 point* | |
| $(= -2 \cdot 3 + 1) = -5$. | 1 point* | |
| **Total:** | **6 points** | |

*Note: The 3 points marked by * may also be given for the following reasoning:*

| | | |
|---|---|---|
| The equation of any line through point $B$, not parallel to the axis of the parabola, may be written as $y = mx - 3m$, where $m$ is the gradient of such line. | 1 point | |
| This line has a single common point with the parabola if the discriminant of the equation $x^2 + (m-1)x - 3(m+2) = 0$ is 0. | 1 point | |
| The discriminant is $(m-1)^2 + 12(m+2) = (m+5)^2$, so the gradient of the tangent is –5. | 1 point | |

## II.

| **5. a)** | | |
|---|---|---|
|  Triangle $ABE$ is right and isosceles, | 1 point | |
| so $ABE\angle = 45°$. | 1 point | |
| Cut a regular triangle of side 2 units along one of its axes of symmetry. The triangles obtained will be congruent to triangle $BCD$ (two sides and the angle between them are equal), | 1 point | *Connect the midpoint of side CB to vertex D. This segment divides triangle BCD into a regular triangle of unit sides and an isosceles triangle.* |
| so $DBC\angle = 30°$. | 1 point | |
| The angle between the two diagonals is therefore: $EBD\angle = ABC\angle - ABE\angle - DBC\angle = 75°$. | 1 point | |
| **Total:** | **5 points** | |

| **5. b)** **Solution 1** | | |
|---|---|---|
| $\cos 75° = \cos(30° + 45°) =$ <br> $= \cos 30° \cdot \cos 45° - \sin 30° \cdot \sin 45° =$ | 1 point | |
| $= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2} =$ | 1 point | |
| $= \frac{\sqrt{6} - \sqrt{2}}{4}$ (and so the statement is proven). | 1 point | |
| **Total:** | **3 points** | |

*Note: Do not accept calculations with approximate figures.*

| **5. b)** **Solution 2** | | |
|---|---|---|
| $\cos^2 75° = \frac{1 + \cos 150°}{2} =$ | 1 point | |
| $= \frac{1 - \frac{\sqrt{3}}{2}}{2} = \frac{2 - \sqrt{3}}{4}$. | 1 point | |
| As $\left(\frac{\sqrt{6} - \sqrt{2}}{4}\right)^2 = \frac{8 - 2\sqrt{12}}{16} = \frac{2 - \sqrt{3}}{4}$, <br> the statement is proven (as $\cos 75° > 0$). | 1 point | |
| **Total:** | **3 points** | |

*Note: Do not accept calculations with approximate figures.*

| **5. c)** | | |
|---|---|---|
| $BE = \sqrt{2}$ | 1 point | |
| $BD = \sqrt{3}$ | 1 point | |
| Apply the Law of Cosines in triangle *EBD*: <br> $DE^2 = 2 + 3 - 2 \cdot \sqrt{2} \cdot \sqrt{3} \cdot \frac{\sqrt{6} - \sqrt{2}}{4} =$ | 1 point | |
| $= 2 + \sqrt{3}$. | 1 point | |
| And so $DE = \sqrt{2 + \sqrt{3}}$. | 1 point | |
| **Total:** | **5 points** | |

| **5. d)** | | |
|---|---|---|
| $\left(\frac{\sqrt{6} + \sqrt{2}}{2}\right)^2 = \frac{8 + 2\sqrt{12}}{4} =$ | 1 point | $\sqrt{2 + \sqrt{3}} = \sqrt{\frac{4 + 2\sqrt{3}}{2}} =$ |
| $= 2 + \sqrt{3} \left( = \left(\sqrt{2 + \sqrt{3}}\right)^2 \right)$ | 1 point | $\sqrt{\frac{(\sqrt{3} + 1)^2}{2}} = \frac{\sqrt{3} + 1}{\sqrt{2}} =$ |
| Both of these numbers are positive and their squares are equal, so the statement is proven. | 1 point | $= \frac{\sqrt{6} + \sqrt{2}}{2}$ |
| **Total:** | **3 points** | |

*Note: Award 0 points if the candidate uses approximate figures.*

| **6. a)** | | |
|---|---|---|
| (1) true<br>(2) false<br>(3) true<br>(4) true<br>(5) false | 3 points | *Award 2 points for 4 correct answers, 1 point for 3 correct answers, 0 points for less than 3 correct answers.* |
| **Total:** | **3 points** | |

| **6. b)** | | |
|---|---|---|
| The statement is false. | 1 point | |
| Any proper counterexample (10-point, simple graph with at least 8 edges, that contains a circuit). | 2 points | |
| **Total:** | **3 points** | |

| **6. c)** | | |
|---|---|---|
| If a (10-point, simple) graph does not contain a circuit then it has at least 8 edges. | 1 point | |
| The converse is false. | 1 point | |
| Any proper counterexample (10-point simple graph with no more than 7 edges that does not contain a circuit). | 2 points | |
| **Total:** | **4 points** | |

| **6. d) Solution 1** | | |
|---|---|---|
| A 10-point complete graph has $\left(\frac{10\cdot 9}{2}=\right)$ 45 edges. | 1 point | |
| Selecting three edges is possible in $\binom{45}{3}$ (= 14 190) different ways (all of which are equally likely). | 1 point | |
| A circuit of three edges means that the endpoints of the three selected edges are exactly three vertices of the graph, i.e. selecting any three different vertices of the graph is equivalent to selecting three edges forming a circuit. | 1 point | |
| The number of favourable cases: $\binom{10}{3}$ (= 120). | 1 point | |
| The probability: $\frac{\binom{10}{3}}{\binom{45}{3}} \approx$ | 1 point | |
| ≈ 0.0085. | 1 point | |
| **Total:** | **6 points** | |

| **6. d)** **Solution 2** | | |
|---|---|---|
| A 10-point complete graph has $\left(\frac{10\cdot 9}{2}=\right)$ 45 edges. | 1 point | |
| The first selected edge may be any one of these. It is necessary, however, that the second selected edge has a common endpoint with the first one. | 1 point | |
| From both endpoints of the first edge there are 8-8 different edges drawn. Select any one of these from the remaining 44 edges. The probability of correctly selecting the second edge is therefore $\frac{16}{44}$. | 1 point | |
| For the third edge, select the one out of the remaining 43 that connects the "free" endpoints of the two edges already selected. The probability of a correct selection is therefore $\frac{1}{43}$. | 1 point | |
| (As the selection of each edge is independent from the selection of the others) the probability we are looking for is $\frac{16}{44}\cdot\frac{1}{43}=$ | 1 point | |
| $\left(=\frac{4}{473}\right)\approx 0.0085$. | 1 point | |
| **Total:** | **6 points** | |

| **6. d)** **Solution 3** | | |
|---|---|---|
| A 10-point complete graph has $\left(\frac{10\cdot 9}{2}=\right)$ 45 edges. | 1 point | |
| Selecting three edges is possible in $\binom{45}{3}$ (= 14 190) different ways (each of which is equally likely). | 1 point | |
| Draw one edge. This is possible in 45 different ways. Draw the edges from the endpoints of the first edge to any of the remaining 8 points of the graph, thereby creating a circuit. This means there are a total $45\cdot 8$ (= 360) circuits of three edges in the graph, | 1 point | |
| every one of which has been counted exactly three times though, therefore the correct number of different circuits of three edges is 120. | 1 point | |
| The probability is: $\frac{120}{\binom{45}{3}}\approx$ | 1 point | |
| $\approx 0.0085$. | 1 point | |
| **Total:** | **6 points** | |

| **7. a)** | | |
|---|---|---|
| With all the given conditions, it is enough to examine the angles of the triangle only. Let $a$ be the middle angle of the triangle and $d$ be the common difference of the arithmetic sequence ($d > 0$). In this case the three angles are $a - d$, $a$, and $a + d$ ($a$ and $d$ are both integers). | 1 point | *This point is also due if the correct reasoning is reflected only by the solution.* |
| $(a - d) + a + (a + d) = 180$, which means $a = 60$ (this is the middle angle of the triangle). | 1 point | |
| Given that the triangle is acute (and $d$ is a positive integer), the largest angle is at least 61°, at most 89°. | 1 point | |
| Therefore, there are 29 different such triangles. | 1 point | |
| **Total:** | **4 points** | |

| **7. b) Solution 1** | | |
|---|---|---|
| The sum of the interior angles (measured in degrees) of such a regular polygon is $n^2$, so $n^2 = (n-2)\cdot 180$ | 1 point | |
| $n^2 - 180n + 360 = 0$ ($n \in \mathbf{N}$ and $n \geq 3$). | 1 point | |
| The solutions of the quadratic equation are not integers (≈ 177.98, and ≈ 2.02). | 1 point | |
| Therefore, there really isn't any such polygon. | 1 point | |
| **Total:** | **4 points** | |

| **7. b) Solution 2** | | |
|---|---|---|
| If one interior angle of a regular polygon is 179° than it has 360 sides (not 179), so $n \neq 179$. | 1 point | |
| If one interior angle of a regular polygon is 178° than it has 180 sides (not 178), so $n \neq 178$. Similarly, if one interior angle of a regular polygon is 177° than it has 120 sides (not 177), so $n \neq 177$. | 1 point | |
| The exterior angle of a regular polygon, $\left(\frac{360°}{n}\right)$, is inversely proportional to $n$. If the measure of the interior angle decreases then (as the exterior angle increases) the number of sides will decrease, too. Similarly, if the number of sides increases, the measure of the interior angle will increase, too. | 1 point | |
| The only possible option is when $n < 120$, but only the regular triangle, the square and the regular pentagon has interior angles below 120° and neither complies with the given conditions. Hence, the statement given is true. | 1 point | |
| **Total:** | **4 points** | |

*Note: Full points may be given if the candidate (with adequate reasoning) lists every possible regular polygon whose angles, in degrees, are integers and thereby correctly argues that the polygon indicated by the question does not exist. (A table that lists possible cases is attached to the solution of question 7c.)*

| **7. c)** | | |
|---|---|---|
| The degree measure of one interior angle of the $n$-sided regular polygon ($n \geq 3$) is: $\frac{(n-2)\cdot 180}{n}$, which is now equal to a certain $k$ positive integer. | 1 point* | *If the degree measure of one interior angle of the regular n-sided polygon is k ($k \in \mathbf{N}^{+}$), then the measure of one exterior angle is 180 – k.* |
| So $nk = n\cdot 180 - 360$, | 1 point* | *The sum of the exterior angles is* 360°, *and so* $\frac{360}{n} = 180 - k$. |
| i.e. $k = 180 - \frac{360}{n}$. | 1 point* | |
| As $n \geq 3$, all positive divisor of 360 that are greater than 2 are possible values of $n$ (as these will all give a positive integer value less than 180 for $k$). | 1 point* | *This point is also due if the correct reasoning is reflected only by the solution.* |
| Divisor-pairs of 360 are: (1; 360), (2; 180), (3; 120), (4; 90), (5; 72), (6; 60), (8; 45), (9; 40), (10; 36), (12; 30), (15; 24), (18; 20). | 2 points** | *As* $360 = 2^3\cdot 3^2\cdot 5$, *the number of positive divisors of 360 is* $4\cdot 3\cdot 2 = 24$. |
| 360 has 24 positive divisors. | 1 point | |
| (1 and 2 are not suitable divisors, therefore) there are 22 different possible values for $n$. | 1 point | |
| **Total:** | **8 points** | |

*Notes:*

*1. The table below contains all possible values of n and the corresponding values of k.*

| $n$ | $k$ (°) | $n$ | $k$ (°) | $n$ | $k$ (°) | $n$ | $k$ (°) | $n$ | $k$ (°) |
|---|---|---|---|---|---|---|---|---|---|
| 3 | 60 | 9 | 140 | 20 | 162 | 40 | 171 | 90 | 176 |
| 4 | 90 | 10 | 144 | 24 | 165 | 45 | 172 | 120 | 177 |
| 5 | 108 | 12 | 150 | 30 | 168 | 60 | 174 | 180 | 178 |
| 6 | 120 | 15 | 156 | 36 | 170 | 72 | 175 | 360 | 179 |
| 8 | 135 | 18 | 160 | | | | | | |

*2. Award 0 points of the 2 marked by ** if the candidate finds no more than 9 divisor-pairs. Award 1 point if the candidate finds 10 or 11 pairs.*

*3. The 4 points marked by * may also be given for the following reasoning:*

| | | |
|---|---|---|
| The degree measure of one interior angle of a regular polygon is an integer if and only if the same is true for the external angle. Let us examine one interior angle of the polygon. | 1 point | |
| The degree measure of one exterior angle of the $n$-sided regular polygon ($n \geq 3$) is $\frac{360}{n}$, | 1 point | |
| The degree measure of one interior angle will be an integer only if $\frac{360}{n}$ is an integer (less than 180), too. | 1 point | |

| | | |
|---|---|---|
| As $n \geq 3$, all positive integer divisors of 360, greater than 2, are possible values of $n$. | 1 point | *This point is also due if the correct reasoning is reflected only by the solution.* |

| **8. a) Solution 1** | | |
|---|---|---|
| The probability that a randomly selected person is not infected is 0.998. | 1 point | |
| $P$(at least 1 person is infected out of the 80) = = 1 – $P$(nobody is infected) = | 1 point | |
| $= 1-0.998^{80} \approx$ | 1 point | |
| $\approx 0.15$. | 1 point | *Do not award this point if the solution is not rounded or rounded incorrectly.* |
| **Total:** | **4 points** | |

| **8. a) Solution 2** | | |
|---|---|---|
| The probability that a randomly selected person is not infected is 0.998. | 1 point | |
| $P$(1 person is infected) = $\binom{80}{1} \cdot 0.002 \cdot 0.998^{79} \approx$ 0.1366. Similarly, $P$(2 people infected) ≈ 0.0108, $P$(3 people infected) ≈ 0.0006. | 1 point | |
| As $P$(4 people infected) ≈ 0.00002 (and further probabilities decrease rapidly) the rest of the terms in the total probability may be omitted which will not influence the final result (within the required accuracy). | 1 point | |
| This gives $P$(at least 1 person is infected out of 80) ≈ ≈ 0.1366 + 0.0108 + 0.0006 ≈ 0.15. | 1 point | *Do not award this point if the solution is not rounded or rounded incorrectly.* |
| **Total:** | **4 points** | |

| **8. b)** | | |
|---|---|---|
| The proportion of those infected within the population of the city grows by a factor of 1.05 each day. | 1 point | *This point is also due if the correct reasoning is reflected only by the solution.* |
| Let $x$ be the number of days we are looking for. In this case $0.2 \cdot 1.05^x = 1$. | 1 point | |
| $1.05^x = 5$ | 1 point | |
| $x = \dfrac{\log 5}{\log 1.05} \approx 32.99$ | 1 point | $x = \log_{1.05} 5$ |
| (As the number of people infected increases) it takes about 33 days until 1% of the population becomes infected. | 1 point | |
| **Total:** | **5 points** | |

*Note: Full score may be awarded if the candidate gives the correct answer by solving an inequality instead of an equation.*

| **8. c) Solution 1** | | |
|---|---|---|
| Let $n$ be the number of people living in the city. | 1 point | *The answer to this question does not depend on the actual population of the city. Let it, for example, be* $n = 1\ 000\ 000$. |
| The number of people infected is $0.002n$,<br>the number of those not infected is $0.998n$. | 1 point | *In this case, there are* 2000 *people infected, and* 998 000 *people not infected in the city.* |
| Assuming everyone took the test, $0.002n \cdot 0.99 = 0.00198n$ people would turn out positive, while in the case of the other $0.00002n$ people the test would turn out negative. | 1 point | *Out of* 2000 *infected people* $(2000 \cdot 0.99 =)$ 1980 *would give positive test results, while the other* 20 *would be negative.* |
| Out of $0.998n$ people not infected<br>$0.998n \cdot 0.04 = 0.03992n$ would give positive results, while the other $0.95808n$ would be negative. | 1 point | *Out of* 998 000 *people not infected,* $(998\,000 \cdot 0.04 =)$ 39 920 *people would give positive test results, while the remaining* 958 080 *people would be negative.* |
| The total number of those with positive test result is $0.00198n + 0.03992n = 0.0419n$, out of whom $0.00198n$ are truly infected. | 1 point | *The total number of those with positive test result is* 39 920 + 1980 = 41 900, *out of whom* 1980 *are truly infected.* |
| $\frac{0.00198n}{0.0419n} \approx 0.0473$, | 1 point | $\frac{1980}{41\,900} \approx 0.0473$ |
| which means the probability of being infected is less than 0.05 despite a positive test result. | 1 point | |
| **Total:** | **7 points** | |

| **8. c)** **Solution 2** | | |
|---|---|---|
| Let $A$ be the event that the test is positive, and let $B$ be the event that the user is infected. The probability $P(B\mid A)$ is to be determined. | 1 point | *This point is also due if the correct reasoning is reflected only by the solution.* |
| The probability that an infected person took the test and it turned out positive is: $P(AB) = 0.002\cdot 0.99$ $(= 0.00198)$. | 1 point | |
| The probability that an uninfected person took the test and it turned out positive is: $P(A\overline{B}) = 0.998\cdot 0.04$ $(= 0.03992)$. | 1 point | |
| (Events $AB$ and $A\overline{B}$ are mutually exclusive, and so the probability that a randomly selected person living in the city is showing positive test results is the sum of the above probabilities:) $P(A) = 0.998\cdot 0.04 + 0.002\cdot 0.99$ $(= 0.0419)$. | 1 point | |
| $P(B\mid A) = \dfrac{P(AB)}{P(A)} =$ | 1 point | |
| $= \dfrac{0.002\cdot 0.99}{0.002\cdot 0.99 + 0.998\cdot 0.04} \approx 0.0473$, | 1 point | $\dfrac{0.00198}{0.0419} \approx 0.0473$ |
| which means the probability of being infected is less than 0.05 despite a positive test result. | 1 point | |
| **Total:** | **7 points** | |

| **9. a)** **Solution 1** | | |
|---|---|---|
| Let $x$ be the mass of one of the parts $(0 < x < 350)$. The mass of the other part is $350 - x$, the total cost of transportation is $k(x) = \dfrac{x^2}{10} + 205 + \dfrac{(350-x)^2}{10} + 205$ (Euros). | 1 point | |
| Rearranged: $k(x) = \dfrac{1}{5}(x^2 - 350x + 63\ 300)$. | 1 point | $k'(x) = \dfrac{1}{5}(2x - 350)$ |
| As $k(x) = \dfrac{1}{5}(x-175)^2 + 6535$ (or $k'(175) = 0$), function $k$ has a minimum when $x = 175$, which makes the statement true. | 2 points | |
| **Total:** | **4 points** | |

| **9. a)** **Solution 2** | | |
|---|---|---|
| Let the mass of one part be $175 - x$ tons ($0 \le x < 175$), in which case the mass of the other part is $175 + x$ tons. The total cost of transportation: $k(x) = \frac{(175-x)^2}{10} + 205 + \frac{(175+x)^2}{10} + 205$ (Euros). | 1 point | |
| Rearranged: $k(x) = \frac{1}{5}x^2 + 6535$. | 1 point | |
| This is minimal when $x = 0$, so the statement is true. | 2 points | |
| **Total:** | **4 points** | |

| **9. a)** **Solution 3** | | |
|---|---|---|
| Let $x$ be the mass of one of the parts ($0 < x < 350$). The total cost of transportation is $k(x) = \frac{x^2}{10} + 205 + \frac{(350-x)^2}{10} + 205$ (Euros). | 1 point | |
| It is minimal when $x^2 + (350-x)^2$ is minimal. | 1 point | |
| Use the relation between the Quadratic and the Arithmetic Means: $x^2 + (350-x)^2 \ge 2 \cdot \frac{(x+350-x)^2}{4} = \frac{350^2}{2}$. | 1 point | |
| The two means are equal when $x = 350 - x$, and so the statement is true. | 1 point | |
| **Total:** | **4 points** | |

| **9. b)** | | |
|---|---|---|
| The cost of transportation by rail: $n \cdot \frac{350^2}{10n^2} + 205n$. | 2 points | *Award 1 point for each term of the sum.* |
| Doing the square and division, it really is equal to $\frac{12\,250}{n} + 205n$. | 1 point | |
| **Total:** | **3 points** | |

| **9. c) Solution 1** | | |
|---|---|---|
| The total cost of transportation (including the separation into parts): $\frac{12\,250}{n}+205n+400(n-1)$ $(n \in \mathbf{N}^{+})$. | 1 point | |
| (Examine function *f* that is defined over the set of positive real numbers, where) $f(x)=\frac{12\,250}{x}+605x-400$. | 1 point | |
| Function *f* is differentiable and $f'(x)=-\frac{12\,250}{x^2}+605$. | 1 point | |
| (Function *f* has an extreme wherever *f'* is zero:) $-\frac{12\,250}{x^2}+605=0$. | 1 point | |
| (As $x > 0$) $x=\sqrt{\frac{12\,250}{605}}$ $(\approx 4.4998)$. | 1 point | |
| As $f''(x)=\frac{24\,500}{x^3}$ $(x \in \mathbf{R}^{+})$ is positive on its domain, the function *f* has an absolute minimum at $\sqrt{\frac{12\,250}{605}}$. | 1 point | *Other reasons are also acceptable, e.g. reference to the change of sign of the first derivative.* |
| As this number is not an integer, (considering the monotonicity of the function *f*) there are two options: the load must be divided into either 4 or 5 parts. | 1 point | *Function f is (strictly) monotone decreasing on* $]0; 4.4997[$, *(strictly) monotone increasing on* $]4.4998; +\infty[$. |
| If $n = 4$, the total cost is 5082.5 Euros, if $n = 5$, the total cost is 5075 Euros. | 1 point | |
| The total cost is lowest possible if the load is divided into 5 parts of equal mass. | 1 point | |
| **Total:** | **9 points** | |

| **9. c) Solution 2** | | |
|---|---|---|
| $f(1) = 12\,455$, $f(2) = 6935$, $f(3) \approx 5498$, $f(4) = 5082.5$, $f(5) = 5075$, $f(6) \approx 5272$. | 2 points | |
| The lowest cost seems to belong to the case when the load is divided into 5 parts. | 1 point | *This point is also due if the correct reasoning is reflected only by the solution.* |
| Examine the inequality $\frac{12\,250}{n}+605n-400<5075$ (that is defined over the set of real numbers). | 2 points | |
| (Because $n > 0$) the above inequality is equivalent to $121n^2-1095n+2450<0$. | 1 point | |
| The solution set of which is $\left]\frac{490}{121}; 5\right[$, | 1 point | |
| However, there are no integers within this interval. | 1 point | |
| The total cost is therefore minimal if the load is divided into 5 parts of equal mass. | 1 point | |
| **Total:** | **9 points** | |